

Элизиум рай не на Земле

$$\sin^6 x + \cos^6 x = \sin x \cdot \cos x$$

1 СПОСОБ
 $((1-\cos 2x)/2)^3 + ((1+\cos 2x)/2)^3 = \sin x \cdot \cos x$
 $(1-\cos 2x)^3/8 + (1+\cos 2x)^3/8 = \sin x \cdot \cos x$
 $[(1-\cos 2x)^3 + (1+\cos 2x)^3]/8 = \sin x \cdot \cos x$
 $[1-3\cos 2x+3\cos^2(2x)-\cos^3(2x) + 1+3\cos 2x+3\cos^2(2x)+\cos^3(2x)]/8 = \sin x \cdot \cos x$
 $[2+6\cos^2(2x)]/8 = \sin x \cdot \cos x$
 $2+6\cos^2(2x)=8\sin x \cdot \cos x$
 $2+6\cos^2(2x)=4\sin 2x$
 $2+6(1-\sin^2(2x))=4\sin 2x$
 $2+6-6\sin^2(2x)=4\sin 2x$
 $8-6\sin^2(2x)=4\sin 2x$
 $\sin 2x=t$
 $8-6t^2=6t$
 $8t^2+6t-8=0$
 $4t^2+3t-4=0$
 $D=9+64=73$
 $t1=(-3+\sqrt{73})/8$
 $t2=(-3-\sqrt{73})/8$
 $\sin 2x=(-3+\sqrt{73})/8$
 $2x=\arcsin((-3+\sqrt{73})/8)+2Pk$
 $x=\arcsin((-3+\sqrt{73})/8)/2+Pk$
 $2x=P-\arcsin((-3+\sqrt{73})/8)+2Pk$
 $x=P/2-\arcsin((-3+\sqrt{73})/8)/2+Pk$

2 СПОСОБ
 $(\sin^2 x)^3 + (\cos^2 x)^3 = \sin x \cdot \cos x$
 $(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cdot \cos^2 x + \cos^4 x) = \sin x \cdot \cos x$
 $(\sin^4 x - \sin^2 x \cdot \cos^2 x + \cos^4 x) = \sin x \cdot \cos x / (1 + \sin^2 x + \cos^2 x)$
 $1 = \sin x \cdot \cos x + 3\sin^2 x \cdot \cos^2 x$
 $1 = \sin x \cdot \cos x + 3\sin^2 x \cdot \cos^2 x$
 $\sin x \cdot \cos x = t$
 $1 = t + 3t^2$
 $3t^2 + t - 1 = 0$
 $D = 1 + 12 = 13$
 $t1 = (-1 + \sqrt{13})/6$
 $t2 = (-1 - \sqrt{13})/6$
 $\sin x \cdot \cos x = (-1 + \sqrt{13})/6$
 $\sin 2x/2 = (-1 + \sqrt{13})/6$
 $\sin 2x = (-1 + \sqrt{13})/3$
 $2x = \arcsin((-1 + \sqrt{13})/3) + 2Pk$
 $x = \arcsin((-1 + \sqrt{13})/3)/2 + Pk$
 $2x = P - \arcsin((-1 + \sqrt{13})/3) + 2Pk$

$$x = P/2 - \arcsin((-1 + \sqrt{13})/3)/2 + Pk$$

Воспользоваться $a^5 + b^5 =$

$$a^3 + b^3 + a^2b - a^2b + ab^2 - ab^2 = (a^3 + a^2b) + (b^3 + ab^2) - (a^2b + ab^2) = a^2(a+b) + b^2(b+a) - ab(a+b) = (a+b)(a^2 - ab + b^2)$$

$$a^5 + b^5 = a^5 + b^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 - a^4b - a^3b^2 - a^2b^3 - ab^4 =$$

$$= (a^5 + a^4b) + (b^5 + ab^4) + (a^3b^2 + a^2b^3) - (a^4b + a^3b^2) - (a^2b^3 + ab^4) =$$

$$= a^4(a+b) + b^4(b+a) + a^2b^2(a+b) - a^3b(a+b) - ab^3(a+b) = (a^4 - a^3b + a^2b^2 - a$$

$$b^3 + b^4)$$

Смотрим на выражение $a^3 + b^3$ как на ф-ию от a , где b - просто число

$$f(a) = a^3 + b^3$$

Ищем (угадываем) корень этой ф-ии $a = -b$

По Т Безу мы можем быть уверены что $a^3 + b^3$ делится на $(a - \text{корень}) = (a - (-b)) = (a+b)$

нацело

$$a^3 + 0 \cdot a^2 + 0 \cdot a + b^3 | a + b \quad a^3 + 0 \cdot a^2 - a^3 - a^2b = -a^2b$$

$$a^3 + a^2b \quad | a^2 - ab + b^2$$

$$-a^2b + 0 \cdot a$$

$$-a^2b - ab^2$$

$$ab^2 + b^3$$

$$ab^2 + b^3$$

$$0$$

$$a^5 + 0 \cdot a^4 + 0 \cdot a^3 + 0 \cdot a^2 + 0 \cdot a + b^5 | a + b$$

$$a^5 + a^4b \quad | a^4 - a^3b + a^2b^2 - ab^3 + b^4$$

$$-a^4b + 0 \cdot a^3$$

$$-a^4b - a^3b^2$$

$$a^3b^2 + 0 \cdot a^2$$

$$a^3b^2 + a^2b^3$$

$$-a^2b^3 + 0 \cdot a$$

$$-a^2b^3 - ab^4$$

$$ab^4 + b^5$$

$$ab^4 + b^5$$

$$\sin^5 x + \cos^5 x = 1$$

1 способ

$$(\sin x + \cos x)(\sin^4 x - \sin^3 x \cdot \cos x - \sin x \cdot \cos^3 x + \sin^2 x \cdot \cos^2 x + \cos^4 x) = 1$$

$$(\sin x + \cos x)(\sin^4 x - (\sin^3 x \cdot \cos x + \sin x \cdot \cos^3 x)) + \sin^2 x \cdot \cos^2 x + \cos^4 x = 1$$

$$(\sin x + \cos x)(\sin^4 x - \sin x \cdot \cos x (\sin^2 x + \cos^2 x)) + \sin^2 x \cdot \cos^2 x + \cos^4 x = 1$$

$$(\sin x + \cos x)(\sin^4 x - \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 x + \cos^4 x) = 1$$

$$(\sin x + \cos x)(\sin^4 x + 2\sin^2 x \cdot \cos^2 x - 2\sin^2 x \cdot \cos^2 x - \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 x +$$

$$+ \cos^4 x) = 1$$

$$(\sin x + \cos x)((\sin^2 x + \cos^2 x))^2 - 2\sin^2 x \cdot \cos^2 x - \sin x \cdot \cos x + \sin^2 x \cdot \cos^2 x = 1$$

$$(\sin x + \cos x)(\sin^2 x \cdot \cos^2 x - 2\sin^2 x \cdot \cos^2 x - \sin x \cdot \cos x + 1) = 1$$

$$(\sin x + \cos x)(1 - \sin^2 x \cdot \cos^2 x - \sin x \cdot \cos x) = 1$$

Нет ничего кроме $(\sin x + \cos x)$ и $\sin x \cdot \cos x \Rightarrow$ замена $\sin x + \cos x = t$

$$\sin^2 x + 2\sin x \cos x + \cos^2 x = t^2$$

$$1 + 2\sin x \cos x = t^2$$

$$\sin x \cos x = (t^2 - 1)/2$$

$$t(1 - (t^2 - 1)^2/4 - (t^2 - 1)/2) = 1$$

$$t(1 - ((t^4 - 2t^2 + 1) - 2(t^2 - 1))/4) = 1$$

$$t(1 - ((t^4 - 2t^2 + 1 - 2t^2 + 2))/4) = 1$$

$$t(1 - ((t^4 - 4t^2 + 3))/4) = 1$$

$$t((4 - t^4 + 4t^2 - 3)/4) = 1$$

$$t((1 - t^4 + 4t^2)/4) = 1$$

$$t(1 - t^4 + 4t^2)/4 = 1 \quad | *4$$

$$t(1 - t^4 + 4t^2) = 4$$

$$t^5 - 4t^3 + 4t = 4$$

$$t^5 - 4t^3 - t + 4 = 0 \quad |$$

$$t^4 + t^3 - 3t^2 - 3t - 4 = 0$$

$$\sin x + \cos x = 1$$

$$\sin x + \cos x = \sqrt{2}[\sin x \cdot 1/\sqrt{2} + \cos x \cdot 1/\sqrt{2}] = \sqrt{2}[\sin(x + P/4) + \cos x \cdot$$

$$\sin(P/4)] = \sqrt{2}[\sin(x + P/4) + \cos x \cdot \sin(P/4)] = \sqrt{2} \cdot \sin(x + P/4)$$

$$\cos y = 1/\sqrt{2}$$

$$\sin y = 1/\sqrt{2}$$

$$y = P/4$$

$$\sqrt{2} \cdot \sin(x + P/4) = 1$$

$$\sin(x + P/4) = 1/\sqrt{2}$$

$$x + P/4 = P/4 + 2Pk$$

$$x = 2Pk$$

$$x + P/4 = 3P/4 + 2Pk$$

$$x = 2P/4 + 2Pk$$

$$x = P/2 + 2Pk$$

Еще доказать что ур-ие $t^4 + t^3 - 3t^2 - 3t - 4 = 0$ не имеет вещественных корней

<https://docs.google.com/drawings/d/169dHNmutzeRH95wWj1xvszJMapnZe9fVqvKhpinwi7A/>

edit

Ответ: $2Pk; P/2 + 2Pk;$

	1	0	-4	0	-1	4
1	1	1	-3	-3	-4	0

2 способ

$$\sin^5 x + \cos^5 x = \sin^2 x + \cos^2 x$$

$$\sin^5 x + \cos^5 x - \sin^2 x - \cos^2 x = 0$$

$$\sin^2 x (\sin^3 x - 1) + \cos^2 x (\cos^3 x - 1) = 0$$

$$\sin^2 x (\sin x - 1)(\sin^2 x + \sin x + 1) + \cos^2 x (\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(\cos^2 x + \cos x + 1) = 0$$

$$(1 - \cos x^2)(\sin x - 1)(\sin^2 x + \sin x + 1) + (1 - \sin x^2)(\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(1 - \cos x)(1 + \cos x)(\sin x - 1)(\sin^2 x + \sin x + 1) + (1 - \sin x)(1 + \sin x)(\cos x - 1)(\cos^2 x + \cos x + 1) = 0$$

$$(1 - \cos x)(1 + \cos x)(\sin x - 1)(\sin^2 x + \sin x + 1) + (1 - \sin x)(1 + \sin x)(1 - \cos x)(\cos^2 x + \cos x + 1) = 0$$

$$[(1 - \cos x)(\sin x - 1)][(1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1)] = 0$$

$$(1 - \cos x)(\sin x - 1) = 0$$

$$1 - \cos x = 0 \quad \sin x - 1 = 0$$

$$\cos x = 1 \quad \sin x = 1$$

$$x = 2Pk \quad x = P/2 + 2Pk$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1) + (1 + \sin x)(\cos^2 x + \cos x + 1) = 0$$

$$(1 + \cos x)(\sin^2 x + \sin x + 1 + \sin x - \sin x) + (1 + \sin x)(\cos^2 x + \cos x + 1 + \cos x - \cos x) = 0$$

$$(1 + \cos x)(\sin^2 x + 2\sin x + 1 - \sin x) + (1 + \sin x)(\cos^2 x + 2\cos x + 1 - \cos x) = 0$$

$$(1 + \cos x)((\sin x + 1)^2 - \sin x) + (1 + \sin x)((\cos x + 1)^2 - \cos x) = 0$$

$$\sin^2 2x + \sin x + 1$$

$$\sin x = t$$

$$f(t) = t^2 + t + 1$$

$$D = 1 - 4 = -3$$

$(1 + \cos x) = 0$ и $(1 + \sin x) = 0$ не может быть 0

$$1 + \cos x >= 0$$

$$1 + \sin x >= 0$$

Ответ: $2Pk; P/2 + 2Pk;$